AI 15/16

1a. how can minimax be extended to deal with chance elements?

Three types of nodes – human player, computer player and chance events

The utility of a chance node with outcomes n1, …, nk is given by p1v1 + … + pkvk where pi and vi respectively are the probability and utility of the ith outcome

For nodes corresponding to moves by computer player and by the opponent , the minimum and maximum are used respectively as in standard minimax

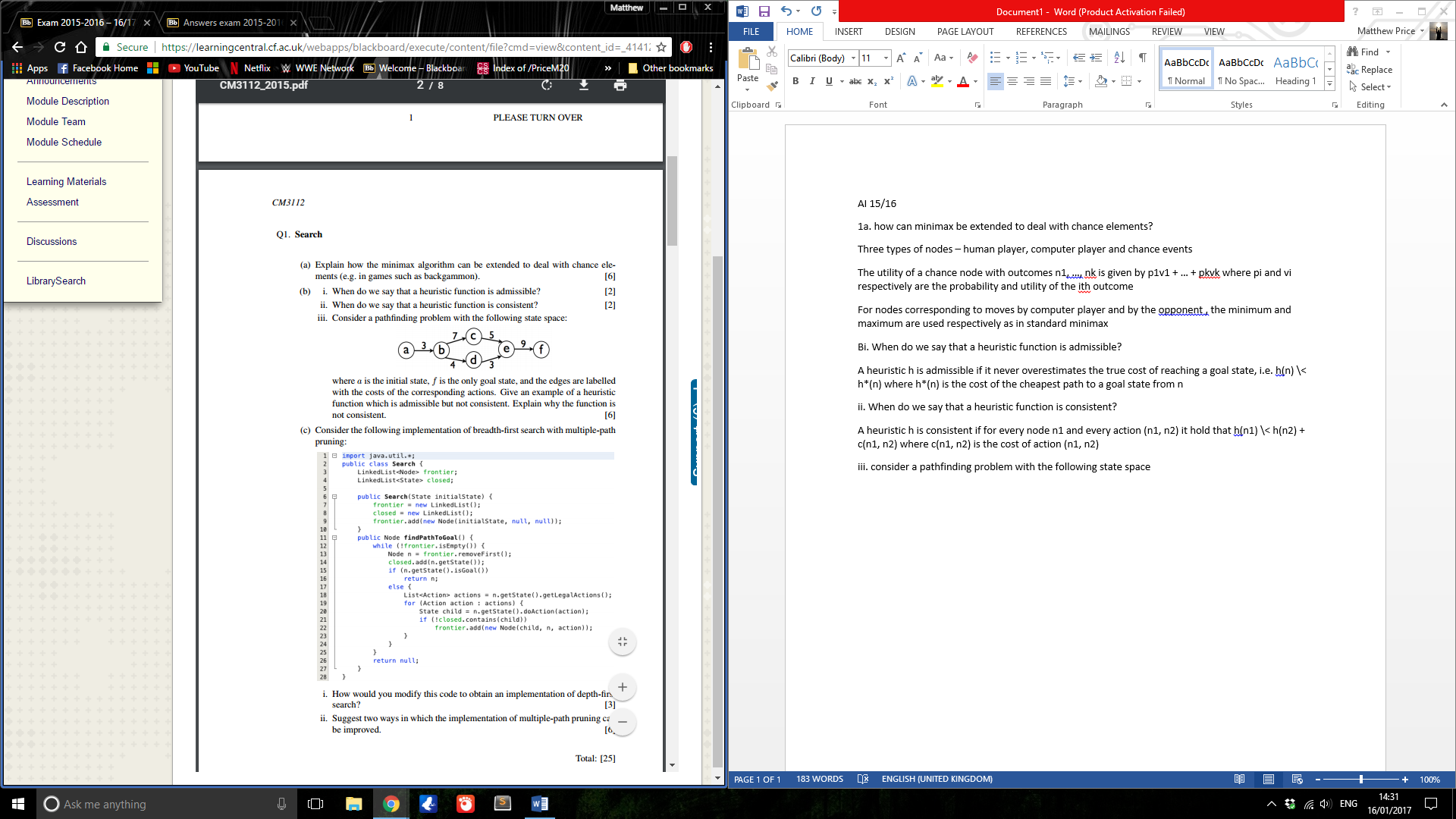
Bi. When do we say that a heuristic function is admissible?

A heuristic h is admissible if it never overestimates the true cost of reaching a goal state, i.e. h(n) \< h\*(n) where h\*(n) is the cost of the cheapest path to a goal state from n

ii. When do we say that a heuristic function is consistent?

A heuristic h is consistent if for every node n1 and every action (n1, n2) it hold that h(n1) \< h(n2) + c(n1, n2) where c(n1, n2) is the cost of action (n1, n2)

iii. consider a pathfinding problem with the following state space



Where a is the initial state, f is the only goal state and the edges are labelled with costs of corresponding actions. Give an example of a heuristic function which is admissible but not consistent, explain why it is not consistent.

A possible example is h(a) = 19, h(b) = 12, h(c) = 14, h(d) = 12, h(e) = 9, h(f) = 0. This heuristic is not consistent because h(a) = 19 > h(b) + c(a, b) = 12 + 3 = 15.

ci. Consider the following implementation of breadth first search with multiple path pruning

how would you modify this code to obtain an implementation of depth first search?

Change removeFirst() to removeLast(), use a stack instead of a LinkedList

ii. suggest two ways in which multiple path pruning can be improved.

The closed list should be a HashSet instead of a LinkedList

Checking if s state occurs in the closed list should be done when expanding the state and not when adding the state to the frontier.

2ai. What is the definition of a possible world?

A possible world is an At -> {true, false} mapping where At is the set of atomic propositions

ii. What is the definition of a tautology?

A tautology is a formula which is satisfied in every possible world.

iii. Explain why a SAT solver can be used to check whether a |= b holds for formulas a and b?

A SAT solver can be used because a |= b holds if a ^¬b is not satisfiable.

b. which of the following formulas entail a or b

2. a or b or c

3. c implies a or b

4. c or not c

6. not a implies ((c or not c) and (not a or d or e) implies b)

c. prove the following entailment relation using the properties of propositional logic that have been provided in the appendix:

e or f implies (a implies b) and (b implies c or d) entails e and a implies c or d

e or f implies (not a or b) and (not b or c or d) implication law

e or f implies (not a or c or d) resolution

not(e or f) or (not a or c or d) implication law

(not e and not f) or (not a or c or do) De Morgan

Not e or not a or c or d weakening

Not (e and a) or c or d De Morgan

E and a implies c or d implication law

3ai. When is it useful to search for partially ordered plans instead of totally ordered plans?

Searching for partially ordered plans is useful when the goal consists of several more or less independent sub goals.

ii. What is the advantage of searching partially – ordered plans instead of totally ordered plans?

The advantage is that the state space can be drastically reduced, as we do not need to specify how the actions in the plans for these sub goals are interleaved

b. Explain how causal links and ordering constraints are used when searching for partially ordered plans

An ordering constraint A ~< B indicates that action A needs to be executed before action B

A causal link A –P> B indicates that the effect p of action A is used as a precondition for B, i.e. that A achieves p for action B.

Ordering constraints are added to prevent actions that have ¬p as an effect to take place between A and B. Specifically if a causal link A –P> B is added, for any action C already in the partial plan which has ¬p as an effect, either the ordering constraint C ~< A or B ~< C is added.

c. Formalize the 8-puzzle as a STRIPS planning problem. Provide a full specification of all actions, as well as the initial final states which are illustrated below:

4ai. What does it mean for random variables to be conditionally independent?

Random variables A and B are conditionally independent given a random variable C if P(A, B|C) = P(A|C) . P(B|C).

ii. What assumptions about conditional independence are encoded by the structure of a Bayesian network?

The network structure encodes the assumption that the value of a random variable X is conditionally independent of its non-descendants, given the value of its parents

b. Give an example of a Bayesian network with random variables A,B,C,D,E such that the Markov blanket of A is given by {B, C}, the Markov blanket of C is given by {A, B, D, E} and the Markov blanket of E is given by {B, C, D}. You only need to provide the network structure not the conditional probability tables.

Markov blanket A, its parents, its children and its childrens parents

c. Calculate the value of P(a, ¬c|b) for P the probability measure defined by the following Bayesian network:

P(a, ¬c|b) = P(a, b, ¬c)/ P(b) by the definition of conditional probability

Using the independence assumptions encoded in the network we find:

P(a,b,¬c) = P(a) . P(b|a) . P(¬c|b)

=0.3 . 0.4 . 0.2

= 0.024

And using the partial theorem

P(b) = P(b|a) . P(a) + P(b|¬a) . P(¬a)

= 0.3 . 0.4 + 0.2 . 0.6

= 0.24

Together we obtain:

P(a, ¬c|b) = 1/10

5ai. What is the definition of the t-conorm?

A t-conorm is a [0,1]^2 – [0, 1] mapping S satisfying for all x,y,z E [0,1]

S(0,x) = x

S(x,y) = S(y,x)

S(x, S(y,z))= S(S(x,y),z)

X \< y -> S(x,z) \< S(y,z)

ii. What is the definition of the residual implicator induced by a continuous t-norm T?

The residual implicator I^T induced by a continuous t-norm T is defined for x,y E [0,1] as:

I^T(x,y) = max{lambda|lambda E[0,1] and T(x, lambda \< y}

b. Let the fuzzy set A and the fuzzy relation R be defined as follows:

A = {a/0.7, b/0.3, c/0.8}

R = {(a, x)/0.9,(a, y)/0.5,(b, x)/0.7,(b, y)/1,(c, x)/0.8,(c, y)/0.6}

Evaluate the direct product of A and R w.r.t the Lukasiewicz t-norm Tw

(A ◦TW R)(x) = max(TW (A(a), R(a, x)), TW (A(b), R(b, x)), TW (A(c), R(c, x)))

= max(TW (0.7, 0.9), TW (0.3, 0.7), TW (0.8, 0.8))

= max(0.6, 0, 0.6)

= 0.6

(A ◦TW R)(y) = max(TW (A(a), R(a, y)), TW (A(b), R(b, y)), TW (A(c), R(c, y)))

= max(TW (0.7, 0.5), TW (0.3, 1), TW (0.8, 0.6))

= max(0.2, 0.3, 0.4)

= 0.4

c. Let A be a set in the universe X and let R be a relation from X to X. The lower

approximation of A w.r.t. R is defined as follows:

A↓R = {x | for every y in X it holds that if (x, y) ∈ R then y ∈ A}

Suggest a way to define the lower approximation of a fuzzy set A w.r.t. a fuzzy

relation R which generalises the above definition

(A↓R)(x) = min y∈X I(R(x, y), A(y)) with I an implicator